8-4 day 1 Improper Integrals

Learning Objectives:

I can evaluate improper integrals with limits of ∞ or $-\infty$

I can test an improper integral for convergence or divergence

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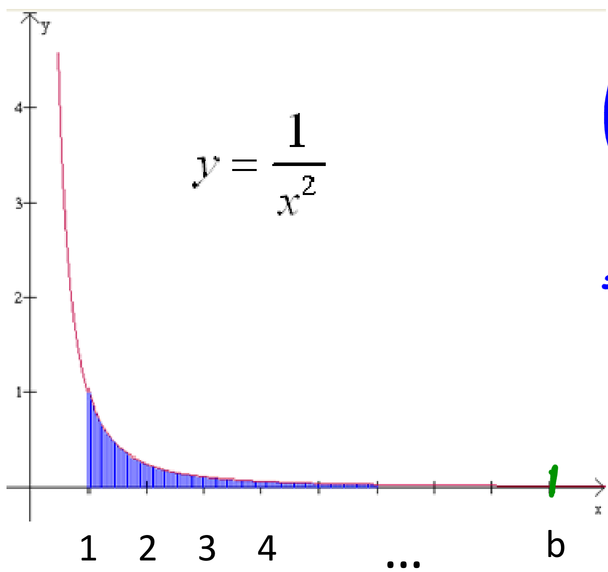


$$\int_1^b \frac{1}{x^2}dx = \int_1^b x^{-2}\,dx$$

$$= -x^{-1}\Big|_1^b = \frac{-1}{x}\Big|_1^b$$

$$= -\frac{1}{b} + \frac{1}{1} = \frac{-1}{b} + 1$$

$$= 1 - \frac{1}{b}$$

$$\int_1^\infty \frac{1}{x^2}dx$$

$$\lim_{b\to\infty} \boxed{\int_1^b \frac{1}{x^2}dx}$$

if $b = 10$ $A = \frac{9}{10}$

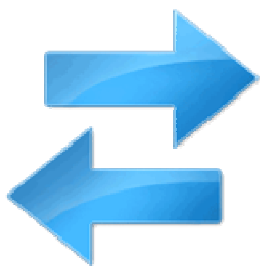
if $b = 100$ $A = \frac{99}{100}$

if $b = 1000$ $A = \frac{999}{1000}$

$\lim_{b\to\infty}$ $A \to 1$

Improper Integral

$$\int_1^{\infty} \frac{1}{x^2}dx = 1$$



$$\lim_{b\to\infty}\int_1^{b} \frac{1}{x^2}dx = 1$$

If $\lim_{b\to\infty}\int_a^b \frac{1}{x^2}dx = L$, then it is said

"the improper integral converges to L"

If $\lim_{b\to\infty}\int_a^b \frac{1}{x^2}dx = \infty$, then it is said

"the improper integral diverges to ∞

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Ex1. Evaluate the integral or state that it diverges.

1.) $\int_1^\infty \frac{1}{x}dx = \lim_{b\to\infty} \boxed{\int_1^b \frac{1}{x}\,dx}$

$= \lim_{b\to\infty} \ln|x| \Big|_1^b$

$= \lim_{b\to\infty} \ln|b| - \ln|1|$

$= \lim_{b\to\infty} \ln|b| - 0$

$= \lim_{b\to\infty} \ln|b| \to \infty$

$\boxed{\text{diverges to } \infty}$

2.) $\int_1^\infty \frac{1}{x^3}dx$

$= \lim_{b\to\infty} \int_1^b x^{-3}\,dx$

$= \lim_{b\to\infty} -\frac{1}{2}x^{-2} \Big|_1^b$

$= \lim_{b\to\infty} -\frac{1}{2x^2} \Big|_1^b$

$= \lim_{b\to\infty} -\frac{1}{2b^2} + \frac{1}{2(1)}$

$= \lim_{b\to\infty} \frac{1}{2} - \frac{1}{2b^2} \to \frac{1}{2}$

$\boxed{\text{converges to } \frac{1}{2}}$

3.) $\int_1^\infty \frac{1}{\sqrt[3]{x}}dx = \lim_{b\to\infty} \int_1^b x^{-1/3}\,dx$

$= \lim_{b\to\infty} \frac{3}{2}x^{2/3} \Big|_1^b$

$= \lim_{b\to\infty} \frac{3}{2}b^{2/3} - \frac{3}{2}(1)^{2/3}$

$= \lim_{b\to\infty} \frac{3}{2}b^{2/3} - \frac{3}{2} \to \infty$

$\boxed{\text{diverges to } \infty}$

4.) $\int_0^1 \frac{1}{x^2}dx = \lim_{a \to 0} \int_a^1 \frac{1}{x^2}dx$

$= \lim_{a \to 0} \int_a^1 x^{-2}\,dx = \lim_{a \to 0} -\frac{1}{x}\Big|_a^1$

$-x^{-1}$

$= \lim_{a \to 0} -\frac{1}{1} + +\frac{1}{a} = \lim_{a \to 0} \frac{1}{a} - 1$

$\infty - 1$

∞

Feb 2-11:20 AM

p-test for Integrals

H.A.	*V.A.*
H HAHH $\int_b^{\infty} \frac{1}{x^p}dx$ H	$\int_0^{b} \frac{1}{x^p}dx$
converges if $p > 1$	diverges if $p \geq 1$
diverges if $p \leq 1$	converges if $p < 1$

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Improper Integrals

1.) If f(x) is continuous on [a, ∞) , then

$$\int_{a}^{\infty} f(x)dx = \lim_{b\to\infty}\int_{a}^{b} f(x)dx$$

2.) If f(x) is continuous on (- ∞,a], then

$$\int_{-\infty}^{b} f(x)dx = \lim_{a\to-\infty}\int_{a}^{b} f(x)dx$$

3.) If f(x) is continuous on (-∞ ,∞)then

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{a\to-\infty}\int_{a}^{b} f(x)dx + \lim_{c\to-\infty}\int_{b}^{c} f(x)dx$$

Where b is any real number

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Ex2. Evaluate the integral or state that it diverges.

1.) $\int_{-\infty}^{0} xe^{x}dx = \lim_{a \to -\infty} \boxed{\int_a^0 xe^x \, dx}$

I by P: $\int u\,dv = uv - \int v\,du$

$u = x \qquad dv = e^x$

$du = 1 \qquad v = e^x$

$\int xe^x dx = xe^x - \int e^x dx$

$= xe^x - e^x$

$= \lim_{a \to \infty} xe^x - e^x \Big|_a^0 = \lim_{a \to -\infty} \left[(0e^0 - e^0) - (ae^a - e^a) \right]$

$= \lim_{a \to -\infty} \left[(-1) \boxed{- ae^a} + e^a \right]$

$-1 + {}^{+}\infty \cdot e^{-\infty} + 0$

$-1 + 0 + 0$

$\lim_{a \to -\infty} ae^a = \lim_{a \to -\infty} \frac{a}{e^{-a}} \quad \frac{-\infty}{\infty}$

$= \lim_{a \to -\infty} \frac{1}{-e^{-a}} = \frac{1}{\infty}$

$= 0$

$\boxed{-1}$

2.) $\displaystyle\int_{-\infty}^{\infty}\frac{1}{1+x^2}dx = \int_{-\infty}^{0}\frac{1}{1+x^2}dx + \int_{0}^{\infty}\frac{1}{1+x^2}dx$

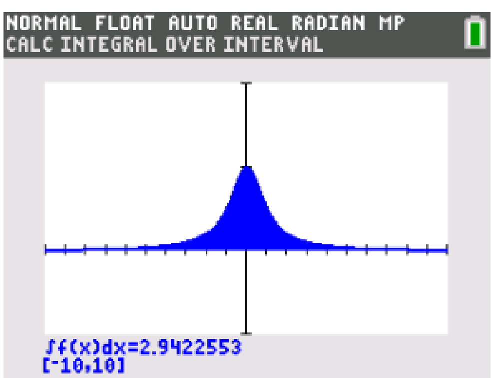
$$=\lim_{A\to-\infty}\int_{A}^{0}\frac{1}{1+x^2}dx + \lim_{B\to\infty}\int_{0}^{B}\frac{1}{1+x^2}dx$$

$$=\lim_{A\to-\infty}\tan^{-1}x\Big|_{A}^{0} + \lim_{B\to\infty}\tan^{-1}x\Big|_{0}^{B}$$

$$=\lim_{A\to-\infty}\left(\tan^{-1}0-\tan^{-1}A\right)+\lim_{B\to\infty}\left(\tan^{-1}B-\tan^{-1}0\right)$$

$$=-\lim_{A\to-\infty}\left(\tan^{-1}A\right)+\lim_{B\to\infty}\left(\tan^{-1}B\right)$$

$$=-\left(-\frac{\pi}{2}\right)+\quad+\frac{\pi}{2}=\boxed{\pi}$$



NORMAL FLOAT AUTO REAL RADIAN MP

$\int_{-100}^{100}(Y_1)dX$

3.12159332

$\int_{-1000}^{1000}(Y_1)dX$

3.139592654

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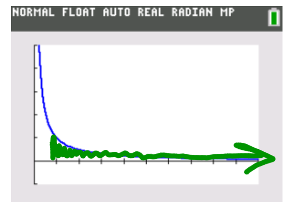
Ex3. Given the function

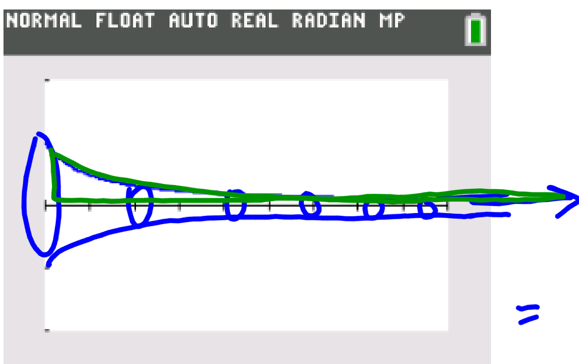
$$f(x) = \frac{1}{x}$$

a.) Find $\int_1^{\infty} f(x)dx$. Explain the meaning of the this integral.

diverge to ∞



b.) The area bounded by f(x), the x-axis, and x=1 is revolved around the x-axis to form a solid. Find the volume of this solid.



πr^2

$$V = \int_1^{\infty} \pi \left(\frac{1}{x}\right)^2 dx$$

$$= \pi \lim_{B \to \infty} \int_1^{B} \frac{1}{x^2} dx$$

$$= \pi \lim_{B \to \infty} \left(\frac{-1}{x}\right)\Big|_1^B$$

$$= \pi \lim_{B \to \infty} -\frac{1}{B} + \frac{1}{1}$$

$$= \pi \lim_{B \to \infty} 1 - \frac{1}{B} = \boxed{\pi}$$

Homework

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